The Effects of Packet Loss on the String Stability of Platoons using Cooperative Adaptive Cruise Control

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ABSTRACT
In the near future, the use of Cooperative Adaptive Cruise Control could contribute to solving the traffic congestion problems experienced in daily life by enabling vehicles to form a platoon, driving with a small distance from each other. This distance should be as short as possible, to optimise throughput on the road. If the distance is too short, however, unexpected actions can cause a traffic jam or even a collision. The distance depends on the characteristics of the network, as the controllers in the vehicles are dependent on the exchange of packets over the wireless medium. We researched the effect of packet loss on the distance by running simulations. Our results indicate that higher amounts of packet loss lead to a worse performance, but the system appears to be resilient to low amounts of packet loss.

Keywords
Cooperative Adaptive Cruise Control (CACC), Markov Chain, Packet Loss, Vehicle-to-Vehicle Communication (V2V)

1. INTRODUCTION
Last century, vehicles were equipped with a controller implementing Cruise Control (CC). This controller enables the vehicle to drive at a constant speed without further user input. Today, controllers implement Adaptive Cruise Control (ACC), extending the capabilities of the CC controller. The ACC controller typically makes use of radar sensors to sense the preceding vehicle, striving to keep a fixed distance from it [6]. The newest innovation in Cruise Control is Cooperative Adaptive Cruise Control (CACC) [9]. The controller implementing CACC extends the ACC controller by using vehicle-to-vehicle (V2V) communications experienced in vehicular ad-hoc networks (VANETS). The communication channel provides the controller with extra information, which enables it to drive closer to the preceding vehicle. A group of vehicles equipped with a CACC controller can drive in close proximity as if they form a train. A group of vehicles close-following each other is called a platoon. Benefits of vehicle platooning are the expansion of lane capacity and a reduction of vehicle drag, thereby reducing fuel usage [8, 2, 3]. In the near future, the use of CACC could contribute to solving the traffic congestion problems experienced in daily life [10]. Research in the field of CACC could also contribute to the development of self-driving vehicles.

The remainder of this paper is organised as follows: Section 1.1 provides an introduction to the most relevant topics of this paper. Related work is discussed in Section 2. Section 3 covers the research methods: the controller of the vehicles, the modelling of the communication channel, a method to compute the expected packet loss ratio of a Markov chain and a simulation road map. The simulation results and discussion are found in Section 4. The conclusion is found in Section 5 and Section 6 provides an overview of future work.

1.1 Problem Statement
1.1.1 Model of the vehicle
We consider a homogeneous platoon of n vehicles. The dynamics of each of the vehicles can be described by:

\[ \ddot{e}_i = -\frac{1}{\tau} a_i + \frac{1}{\tau} u_i, \quad i \in \{1, \ldots, n\} \]  

(1)

where \( d_i \) is the distance between the \( i - 1 \) and \( i^t \) vehicle and \( v_i, a_i, u_i \) are the velocity, acceleration and input of the \( i^t \) vehicle respectively, \( \tau \) represents a time constant for the driveline dynamics. The controllers in the vehicles in the platoon sense their preceding vehicle and communicate with its controller by means of wireless communications. This scenario is depicted in Figure 1.

The spacing error \( e_i(t) \) of a vehicle is defined as:

\[ e_i(t) = d_i(t) - d_{i-1}(t) \]

\[ = d_i(t) - (r_i + hv_i(t)), \quad i \in \{1, \ldots, n\} \]  

(2)

where \( d_{i-1}(t) \) is the desired distance from vehicle \( i - 1 \) to vehicle \( i, r_i \) is the standstill distance between vehicles \( i - 1 \) and \( i \) and \( h \) is the time headway. The time headway is the time it takes a vehicle to cover the distance between its front and the back of the preceding vehicle, \( h \) is the same for every vehicle since we assume a homogeneous platoon. To achieve that the error \( e_i(t) \) asymptotically go to zero, a digital controller is employed.

1.1.2 String stability
The use of the aforementioned controller achieves stability for each individual vehicle, i.e. the error \( e_i(t) \) asymptotically goes to zero, but for safety and comfort purposes, a certain degree of "smoothness" for the trajectories is required. In order to do so, we define string stability as follows:

\[ \|y_i(t)\| \leq \|y_{i-1}(t)\|, \quad i \in \{2, \ldots, n\} \]  

(3)

\[ \|y_i(t)\| \leq \|y_{i-1}(t)\|, \quad i \in \{2, \ldots, n\} \]  

(4)
where $u_r$ is the reference input provided by the leader vehicle and $y_i$ is a linear combination of the state variables of the system of the $i^{th}$ vehicle. Condition (4) assures that the effect of disturbances disappears along with the string of vehicles. Suppose a vehicle in the platoon brakes unexpectedly: succeeding vehicles also have to brake to prevent a collision. If the platoon is string stable, the intensity of braking diminishes downstream. If the platoon is string unstable, the intensity of braking grows instead, possibly causing a traffic jam or even a collision.

### 1.1.3 CACC over unreliable networks

Wireless communication allows for lower values of $h$, increasing the overall performances of the system, e.g. the throughput of the road, but a challenge arises when the network is unreliable. CACC is prone to experience an unstable behaviour when communication packets are lost, therefore, the choice of $h$ must take into account the characteristic of the network. Due to the random nature of packet loss, analytical analysis of the platoon by using (4) is infeasible. Therefore results are obtained by simulation. A packet loss model needs to be developed to mimic the network properties of VANETs [1].

### 1.1.4 Critical time headway

After defining when a platoon is string stable, we can search for the critical time headway, i.e. the lowest value of the time headway for which a platoon is string stable and continues to be string stable for increasing time headways. This critical value is assumed to depend on the quality of the network: if the network experiences a high packet loss ratio the critical time headway of the platoon increases.

### 1.2 Research Questions

1.2.1 What is the effect of packet loss on the critical time headway of platoons?

1.2.2 Is it always possible to find a Bernoulli model equivalent to a Markov chain model?

1.2.3 What is the effect of different sizes of Markov chains on the string stability of platoons?

### 2. RELATED WORK

Network performance is influenced by packet loss and network delays. The effect of packet loss on the string stability of vehicle platoons has been studied using a Bernoulli model to simulate packet loss [7]. We model the network using Markov chains, providing a higher degree of freedom to tune parameters.

### 3. RESEARCH METHODS

The research is conducted using the MATLAB/Simulink software. The simulations consider a platoon of 50 identical vehicles, following a leader vehicle. All vehicles start at a velocity of 0 m/s and the leader vehicle accelerates to 12 m/s. Due to the linearity of the model of the vehicle, running simulations at 12 m/s suffices when determining string stability. The differences between simulation runs are the model of packet loss and the time headway. Plots of the spacing error are generated for every run. These plots are inspected to determine the critical time headway for every model of packet loss. Figures 2 and 3 show the behaviour of a platoon that is string stable and string unstable respectively.

### 3.1 The controller

The controller of the vehicles is designed using the classical feedback theory. The vehicle input is computed by the controller as a differential function of the error (2) and the transmitted packets from the preceding vehicle:

$$u_i(t) = f(e_i(t), e_i'(t), \ldots, u_{i-1}(t), u_{i-1}'(t), \ldots).$$

The controller is parametric: the first simulations are used...
Figure 3. The general behaviour of a string unstable platoon of 50 vehicles.

Figure 4. The general structure of the used Markov chains.

to find a suitable parameter to ensure string stability. This parameter influences the performance of the controller. The used controller is not optimal, as it is not designed using optimisation techniques. Therefore, we are free to choose the value of the parameter, and cannot assure the controller has an optimal performance.

3.2 The communication

A communication model has been developed to simulate packet loss using discrete-time Markov chains [5] where every state of the chain has its own probability of packet loss. Every state is connected solely by its two neighbours, or in case it is an outermost edge, its neighbour and itself, see Figures 4 and 5. $p_{lr,i}$ denotes the packet loss probability in state $i$. After transmitting a packet, one of the edges is chosen at random to transfer to a neighbour state. The choice of leaving out specific edges has been made to reduce simulation times. The Bernoulli model is modelled using this generic Markov chain by using only one state, see Figure 6.

3.3 Expected packet loss ratio of a Markov chain

Here we show how to compute the expected packet loss probability of the used Markov chains. We consider a Markov chain $m$ with three states: it has a space state $S_m = \{1, 2, 3\}$. $p_{lr,i}$ is the probability of packet loss in state $i, i \in S_m$. The transition matrix of $m$ is:

$$M = \begin{bmatrix} 1 - p_1 & p_1 & 0 & \cdots & 0 & 0 \\ 1 - p_2 & 0 & p_2 & \cdots & 0 & 0 \\ 0 & 1 - p_3 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 - p_{n-1} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 - p_n \end{bmatrix}$$

which is a less general case of the matrix depicted in Figure 5. We consider the initial distribution:

$$\hat{p}_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T$$

where the probability of being in state 1 is 1 and the probability to be in state 2 or state 3 is 0. The distribution after 1 transition can be computed by multiplying $\hat{p}_0$ by the transition matrix in (5):

$$\hat{p}_1 = \hat{p}_0^T M = \begin{bmatrix} 1 - p_1 & p_1 & 0 \\ 1 - p_2 & 0 & p_2 \\ 0 & 1 - p_3 & p_3 \end{bmatrix}$$

This process can be repeated indefinitely. Therefore, $\hat{p}_k$ is obtained by multiplying $\hat{p}_{k-1}$ with the transition matrix in (5):

$$\hat{p}_k^T = \hat{p}_{k-1}^T M.$$  

After a certain amount of transitions, $\hat{p}$ is stable, meaning that the distribution no longer changes after a transition. This is denoted by $\pi$, the stationary distribution:

$$\pi^T = \pi^T M.$$
The expected packet loss probability of \( m \) is:

\[
E = \pi^T \begin{bmatrix} p_{lr,1} & p_{lr,2} & p_{lr,3} \end{bmatrix}\, T.
\]  

(10)

Using (5) and (9) we obtain:

\[
\begin{align*}
\pi_1 &= \frac{1 - p_2}{(1 - p_2) + \frac{p_1 p_2}{1 - p_3} + p_1} \\
\pi_2 &= \frac{1 - p_2 p_3 + p_2}{p_1 + p_3} \\
\pi_3 &= \frac{(1 - p_2)(1 - p_3) + p_2 + (1 - p_3)}{p_1 + p_3}.
\end{align*}
\]  

(11) (12) (13)

(10) through (13) are used to construct specific Markov chains with a specific expected packet loss ratio to compare their performance with a Bernoulli model having an equal expected packet loss.

### 3.4 Simulation Road Map

To limit time consumption when simulating, determining the critical time headway is done in multiple iterations with increasing precision. Generated plots of the spacing errors are examined to find bounds on the critical time headway. The lower and upper bounds are the time headway values of the plots before and the plots after the breaking point between a string unstable and a string stable platoon respectively.

#### 3.4.1 Preliminary bounding of the critical time headway

We consider the critical time headway for a platoon without communication and a platoon with perfect communication with a precision of 0.01 seconds. Perfect communication means that all packets are correctly delivered. The results give an upper and lower bound on the critical time headways for different communication models.

#### 3.4.2 Narrow bounding of the critical time headway

Different communication models are tested: a coarse-grained search is performed to find narrower bounds of the critical times headway values. The simulations run the different models for every time headway within the preliminary bounds with a precision of 0.1 seconds. Due to the stochastic nature of packet loss, each simulation is run 10 times, and its results are taken and averaged to study the string stability for a given simulation parameter set. The same simulations are run again with the narrower bounds obtained above and a precision of 0.01 seconds. Simulations are run 20 times. The lowest time headway for which the platoon is string stable is considered the critical time headway.

### 4. SIMULATION RESULTS

Table 1 shows the results of the simulations. For models with an expected packet ratio above 60%, every simulation has been run 40 times since 20 runs proved to be insufficient to get a clear view on the average behaviour of the platoon.

We use the results of models 1 through 9 to look at the effect of packet loss on the critical time headway of platoons, see Figure 7. The performance of the system decreases as the packet loss increases, which is expected. The system appears to be resilient to low amounts of packet loss but drastically performs worse for higher occurrences of packet loss. One should notice that the digital controller used in this research is parametric and not optimal. Therefore, the found critical time headways are not guaranteed to be optimal.

To assess whether a Markov chain can be simplified using a Bernoulli model with an equal expected packet loss ratio, we compare the performance of the different models with identical expected probabilities of packet loss. Models 5, 10 through 15, 23 and 24 have identical expected probabilities of packet loss and have a near-equal performance. Models 6 and 17, 8 and 19 also follow this pattern. Models 20, 21 and 22, however, have the same expected packet loss ratio as models 6, 8 and 9 respectively, but perform worse. Therefore, Markov chain models generally cannot be replaced by a Bernoulli model with an equal expected packet loss ratio. We believe that it is not a coincidence, however, that certain models having identical expected packet loss ratios perform near-equally.

We compare the structure of the Markov chains from the different model and notice that the models that exhibit the observed phenomenon are symmetric about the middle state.

\[
p_{lr,n+1} = E, \\
p_{lr,n+1} - p_{lr,n+1-i} = p_{lr,n+1+i} - p_{lr,n+1}, \quad \forall i \in \{1, \ldots, n\}
\]

with 2n+1 being the number of states of the Markov chain, \( n + 1 \) being the middle state of the Markov chain and \( E \) being the expected packet loss ratio of the Markov chain. More research on this topic is needed to give conclusive results.

To assess the effect of the size of the Markov chain on string stability, the performances of models 5, 15, 23 and 24 are compared. Since these models exhibit the phenomenon described above, these models provided limited insights on the effects of using more complex models to simulate the network.

For determining the string stability of a platoon, simulations were run multiple times to get a more representative overview of the behaviour of the platoon. These results were averaged. Averaging the spacing errors might hide unstable behaviour, as the found critical time headway proves string stability only for the average case. These critical values should, therefore, be considered a lower bound on the actual critical value. An extended definition of string stability to cope with stochastic systems is

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Figure 7. Relation between packet loss ratios and critical time headways.
<table>
<thead>
<tr>
<th>Model</th>
<th>States</th>
<th>Packet loss ratios (%)</th>
<th>Transition ratios (%)</th>
<th>Expected packet loss ratio (%)</th>
<th>Critical time headway (s)</th>
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<td>50 50 50 50 50 50 50 50 50 50</td>
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</table>
missing, but it appears natural that a necessary condition is that the average case is string stable.

5. CONCLUSION
We used Markov chains to model the communication uncertainties in VANETs. Using simulations, we looked at the string stability when using different models. We have shown that the performance of the system, represented by the critical time headway, is resilient to low amounts of packet loss, but degrades substantially for higher amounts.

We found out that a Markov chain cannot be simplified to a Bernoulli model using the expected value of the packet loss ratio, but believe to have found specific properties for which it can. Further research is needed to give conclusive results.

Due to an unfortunate selection of the sample Markov chains, we got limited insights on the effect of using different sized Markov chains.

6. FUTURE WORK
The digital controller used in this research is parametric and not optimal. Therefore, the found critical time headways are suboptimal. Further research should find the optimal controller to achieve a better performance. Also, the controller uses the information of the previously received packet if a packet is lost. This is a fine solution, but different structures can be employed. For instance, a predictor can be designed to get an estimate of the lost information. Such predictor might improve the performance of the system.

For this research, only Markov chains of the form described in Figures 4 and 5 were used, in order to reduce simulation time. The effects of different Markov chains need to be researched as more general Markov chains give an increased degree of freedom for modelling the network. Also, the Markov chains used in this research were used to model the network, but it is possible to use Markov chains to model the bursts of packets in the network, using the states to represent the number of consecutive losses [4].

Due to the random nature of the loss process, in the worst case, every packet is lost. If the controller has to take the worst case into account it is effectively equal to a controller implementing ACC, since the network cannot be used. For CACC to have an impact on the performance of the system and still be safe, it is necessary to extend the definition of string stability to cover the more general stochastic case.

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8. REFERENCES